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Preface

Multifield theories: an introduction

Multifield theories in continuum mechanics are called upon to respond to diverse circumstances and needs. Even at a gross glance, one realizes that bodies are composed by a pervasive crowd of patches and that the arrangement of these patches may influence the gross mechanical behavior of the body.

In many important chapters of classical continuum mechanics, of each patch, only the place x it occupies at each time t need be rendered explicit and then the crowding or shearing within the immediate neighborhood of each place need be evaluated. But, ever more frequently, an increasing host of deeper details must be brought to bear upon an adequate picture of the behavior of solids and structures.

For instance, in polymers, the orderly arrangement of long molecules within each patch has an influence on the gross behavior. In that case, a field of directions $\mathrm{d}(x)$ needs be added as a relevant descriptor at each place (perhaps a unit vector, but irrespective of orientation). The molecules might have shapes more complex than little straight sticks or the sticks may not be so perfectly aligned, even locally. Then, a tensor field $\mathscr{N}(\mathbf{x})$ may be involved (perhaps appropriately normalized by the requirement of having null trace). Alternatively, the granulosity of each patch may exceed the molecular level and be relevant at the scale of crystallites or of larger and looser grains (as in sands). Even if, at a rough glance, the patch appears as uniform, it may reveal lacunae at a deeper look, as in porous materials; then, the void fraction field may be invoked and be even insufficient. Some measure of the texture may be needed as in the case of microcracked solids.

A classification of the diverse physical phenomena at a deeper level is a daunting task and so is the choice, within the wealth of mathematical tools available, of that tool which is appropriate to achieve the model adequate to the particular circumstance envisaged. Notwithstanding these difficulties, here we offer the reader an anthology of new results, with the intent to encourage further debate on the outstanding profound issues. As all the chapters are still in the making, and most are far from distilled, some readers might find certain pages a heavy burden. However, when the underlying physics is subtle, the analysis required is far from trivial, though every effort is expended to avoid unnecessary complexities.

A large class of multifield theories encompass materials with microstructures modeled by holonomic order parameters (Cosserat's materials, micromorphic materials, porous solids, liquid crystals, certain elastic structures like plates, shells, etc.). Within this class, a complete placement of the body is described by associating to each patch both its position in the Euclidean space $\in$ and some information about its material texture, the latter specified through an order parameter taking value on a finite-dimensional differentiable manifold $\mathscr{M}$. Thus, the choice of the manifold corresponds to a choice of the geometrical model of the microstructure. Then, balance equations can be obtained directly through the study of macro- and micro-interactions, when the latter can be meaningfully expressed, or by the requirements that the overall power vanishes for every choice of the velocity fields and that the internal power alone vanishes under rigid body motions.

Each property attributed to $\mathscr{M}$ (e.g. the existence of a metric and of a connection) must be justified on the basis of physical evidence. In some instances, a physically significant connection may not be available for $\mathscr{M}$; then gradients and divergences do not seem to have per se absolute meaning, depending, as they do, on the choice of the connection and the possibility to express meaningfully the total interactions between two sub-bodies seems to evade us. Such seems to be the case when non-local actions are present and the

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environment where the sub-bodies exist has a paramount influence. Non-locality is a difficult concept to muster except when non-locality is, in some precise sense, weak. Then, balance equations exact to the second order with respect to some appropriate parameter take an apparently standard form, but zero-order and first-order terms have no independent physical meaning (being terms of a development); only their sum has the necessary invariance properties.

The manifold $\mathscr{M}$ seems to have very modest, physically significant qualitative properties also when its elements are used to specify the value of an internal variable. Then the evolution equation is not a dynamic balance equation; it expresses a kinetic or chemical or thermodynamic property. Internal variable models of damage may lead to ill-posed problems when strain-softening occurs; regularization may be achieved by a more complete multifield description.

Even more delicate is the modeling of stochastic phenomena. If the order is only partial, averages may offer a sufficient description. Alternatively, distribution functions may be invoked; then the additional field takes values in a function space, though there may be situations when all distributions belong to a very special class (say, they are canonical), a degenerate case, again finite dimensional. Anyway, one is confronted with the consequent question of the choice of possible transition rules (usually temperature dependent) from perfect order to a chaotic state; the macroscopic evolution may be itself stochastic.

If the manifold is non-linear, an embedding of $\mathscr{M}$ in a linear space may be a necessary prerequisite to useful developments. Generally, such embedding is only a convenient mathematical device, e.g. for the purpose of adapting Noll's results on the interactions between sub-bodies (interactions which much be valued on a linear space) and one should not err by attributing to $\mathscr{M}$ some geometrical properties of the linear space in which the embedding is made. Because the embedding is not unique, one also faces sometimes the problem of a physically significant choice.

Curiously, the crystalline structure of a solid body is classified among microstructures only when more than one phase is present and circumstances permit, or even force, phase-transitions. Permanence, in general, allows latency and transference of consequences to the study of constitutive relations. Another exception occurs when the crystal order is not perfect and dislocations appear. Then lack of material uniformity generates microstructures; the body may be considered as a non-Riemannian manifold and gauge-field theories find application.

Sometimes, internal constraints binding the order parameters to the gross deformation gradients makes the microstructure latent. The multifield theory reduces to a second (or higher order) gradient theory that is compatible with the Clausius–Duhem inequality.

So far, in this introduction, the model of a body patch is intended to be a neighborhood of a point in the Euclidean space. In a generalized approach, using Finslerian geometry, a body patch is modeled as an open set in $\in \times \mathscr{M}$; physical reasons for this generalization have been variously suggested, though the link with parallel multifield theories is still wanting.

Causes of appearance of microstructures are, in certain circumstances, procedural: e.g., problems involving phase-transitions are often characterized by non-convex energies and to solve the associate variational problem by direct methods, Young measures need to be introduced; these are probability measures in the space of deformation gradients which allow one to obtain local minima describing strong oscillations of the deformation gradient. Now, these non-convex variational problems can be regularized by using an order-parameter approach. Note that some non-convex variational problems concerning localization of deformations and strong oscillations cannot be satisfactorily treated using Young measures: more complicated tensor valued measures, namely H-measures, need to be introduced. The equilibrium of cracked or microcracked bodies can be studied in the form of a free-discontinuity variational problem seeking a natural solution within a space of functions with bounded variation. Fabric tensors, as new order parameters, can be tools in the solution.

Other topics of great interest (both applicative and theoretical) are creation and destruction of microstructures as they occur in plasticity , microsuperplasticity and microcracks, in the growth of bodies and the kinetic roughening of growing surfaces, the fractal growth of surfaces, thin film deposition, surface corrosion, etc.

Problems related to the occurrence of microstructures due to kinetic aggregation of particles at the patch level have their seminal ideas in Boltzman kinetic theory of gases.

Finally, another important and vast class of multifield theories arises from the need to describe bodies which are convenient, mathematically, to think of per se as limits of sequences of standard bodies (and not, cursorily, as tools in a variational procedure). Traditionally, one is satisfied with classes of bodies closed under operations of join and meet, but that those classes are not wide enough is shown by studies on the fractal character of rock boundaries and cracks and of catalyzers, on fingering in multiphase bodies and on the searches (already mentioned above) for the optimal shape of bodies, slender but efficient in supporting loads. Then multifield theories become of the essence: fields of presence (to allow finely distributed cavities or finely distributed inclusions) and of texture are required.

Thus, there is a rich crop of problems concerning materials, relevant to engineering practice, which need schemes richer than the classical one. These problems will be fruitful germs of researches in Mechanics in the near future. In a recent paper, J. Ball argued that they might lead also to the creation of new chapters in mathematical analysis.

G. Capriz
Dipartimento di Matematica
Università di Pisa
via Buonarroti 2
56127 Pisa, Italy
Tel.: +39-050-844218
Fax: +39-050-844224
e-mail: capiz@dm.uniji.it

P.M. Mariano
Dipartimento di Ingegneria
Strutturale e Geotecnica
Università di Roma "La Sapienza"
via Eudossiana 18
00184 – Roma, Italy
Tel.: +39-06-44585276
Fax: +39-06-4884852
e-mail: mariano@scilla.ing.uniromal.it